

Post-Hoc model Approximation with Logic

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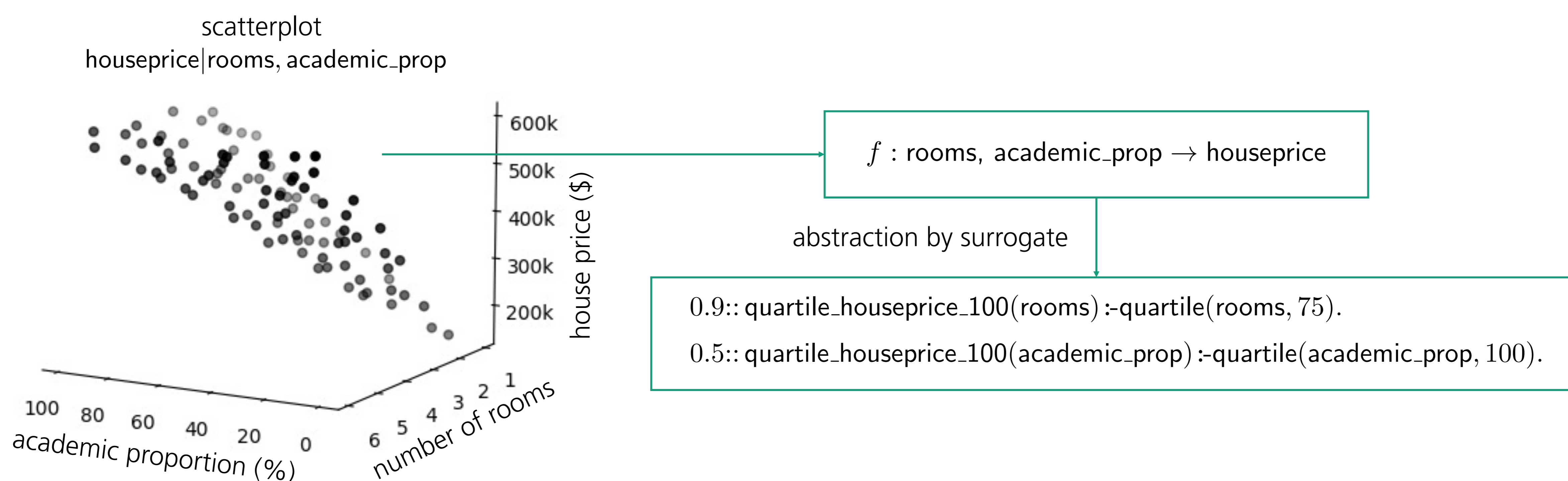
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Motivation

- **Problem:** Regression models are generally built upon domain-specific expert knowledge. Their **ex-post extension by domain experts** with novel knowledge is often infeasible due to the models' complexity.
- **Solution:** We symbolize regression models with **probabilistic logic**. The resulting surrogate model actively considers user preferences.
- **Contribution:** We **reduce the barriers to induce knowledge** into regression models and show that our method is superior to the state of the art.

Approach

- **Goal:** Find a **probabilistic logical surrogate model**.
- **Abstraction step:** Reduce the level of measurement of features and regression target by user-defined **statistical feature extraction** procedures.
- **Rule learning step:** **1)** Convert extracted features into probabilistic logical **predicates**. **2)** Construct **examples** consisting out of abstracted features and target pairs. **3)** Apply the rule learner **ProbFOIL+**.



Example 1. Consider Charlestown (Boston) as a fictive example with an average house price of \$750,000 and an average of 4 rooms per house. Let us say that in Charlestown, the house price is caused by the room number with a probability of 90%, which can be expressed in probabilistic logic as:

0.9 :: houseprice(Charlestown, \$750,000) :- rooms(Charlestown, 4).

Example 2. Assume that we cannot capture whether an average of 4 rooms per house is high or low. We abstract the scalar 4 into its corresponding quartile value by: $\text{quartile}(\text{rooms}(\text{Charlestown})) = 75$, which means that 75% of all districts in Boston on average have 4 or less rooms per house.

Definition 1 (Inverse Coefficient of Variation). Let the *inverse coefficient of variation* (inverse *cv*) approximate the probability of y to be true. Then, $y_{prob} = 1 - cv$, where $cv = y_{cov} \times (y_{mean} + l)^{-1}$ with $l > 0$ and $cv = [0, 1]$.

Example 3. We set $l = 0.01$ to account for divisions by 0. We know from Ex. 1 the average house price of Charlestown, which is \$750,000. Assume for now that it originates in a GPR with $y_{cov} = \$375,000$. Inserting the numbers into the equation of Def. 1 yields: $y_{prob} = 1 - (375,000 \times (750,000 + 0.01)^{-1}) \approx 0.5$.

Example 4. Let us translate the average room number of Charlestown, the only entry in \mathcal{X}_{Exp} , into ProbLog syntax: $t(x_1^{(1)}) = p_{1.1}$. The feature name is retrieved by: $\text{name}(x_1) = \text{rooms}$. Referencing Ex. 2, we straightforwardly formulate: $\text{quartile}(p_{1.1}, 75)$. We specify the target predicate $\text{quartile_houseprice_100}$ and assume that it is satisfied by Charlestown. Recycling y_{prob} from Ex. 3 gives:
0.5 :: quartile_houseprice_100(p_1.1).

Evaluation

Table 1: Surrogate quality. We highlight Welch's tests with $p \leq 0.05$ (m^*).

Data	Method	$\Delta\text{-}fp$	$\Delta\text{-}fn$	# Rules	\emptyset Features	S
Housing	PHAL	0.04 (0.03)	0.11* (0.09)	7.20 (0.84)	2.87 (0.51)	0.85* (0.04)
Housing	GridEx	0.09 (0.05)	0.39 (0.40)	5.00* (0.00)	1.40* (0.55)	0.19 (0.01)
Wine	PHAL	0.04 (0.02)	0.07 (0.04)	2.80* (0.84)	3.68 (0.29)	0.87* (0.06)
Wine	GridEx	0.09 (0.05)	0.47 (0.20)	4.40 (1.14)	1.40* (0.89)	0.26 (0.03)

- **Strategy:** Comparison of PHAL to GridEx for Housing and Wine data sets wrt. fidelity ($\Delta\text{-}fp$, $\Delta\text{-}fn$), complexity (# Rules, \emptyset Features), and stability (S).
- **Results:** Whereas the surrogate's complexity does not reveal a clear trend, PHAL is **superior in terms of fidelity and stability**.

Outlook

- Enrich PHAL with the ability to return **continuous values**.
- Conduct further experiments with several **feature extraction** procedures.
- Apply PHAL to **parameterization** in the automotive sector.
- Exploit PHAL to develop **explainable and interactive training** procedures for regression models.

¹ Luc De Raedt and Angelika Kimmig. Probabilistic (logic) programming concepts. *Mach. Learn.*, 100(1):5-47, 2015.

² Luc De Raedt, Anton Dries, Ingo Thon, Guy Van den Broeck, and Mathias Verbeke. Inducing Probabilistic Relational Rules from Probabilistic Examples. In Qiang Yang and Michael J. Wooldridge, editors, *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015, Buenos Aires, Argentina, July 25-31, 2015*, pages 1835-1843. AAAI Press, 2015.

³ Federico Sabbatini, Giovanni Ciatto, and Andrea Omicini. GridEx: An Algorithm for Knowledge Extraction from Black-Box Regressors. In Davide Calvaresi, Amro Najjar, Michael Winiko, and Kary Främling, editors, *Explainable and Transparent AI and Multi-Agent Systems - Third International Workshop, EXTRAAMAS 2021, Virtual Event, May 3-7, 2021*, Revised Selected Papers, volume 12688 of *Lecture Notes in Computer Science*, pages 18-38. Springer, 2021.

⁴ Christian Wirth, Ute Schmid, and Stefan Voget. Humanzentrierte Künstliche Intelligenz: Erklärendes interaktives maschinelles Lernen für Effizienzsteigerung von Parametrierungsaufgaben. In Ernst A. Hartmann, editor, *Digitalisierung souverän gestalten II*, pages 80-92. Berlin, Heidelberg, 2022. Springer Berlin Heidelberg.